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Theoretical Estimation to the Cyclic Strength Coefficient and the Cyclic Strain-Hardening Exponent for Metallic Materials: Preliminary Study

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The ultimate purpose of the present article is to theoretically estimate the cyclic strength coefficient and the cyclic strain-hardening exponent. For this purpose, the performance parameters of 22 alloys were examined and equations that relate the cyclic strength coefficient and the cyclic strain-hardening exponent to the monotonic tensile ones were developed. Then, by using formulas that express the strength coefficient and the strain-hardening exponent through the four conventional tensile performance parameters, i.e., the yield strength, the ultimate tensile strength, the fracture strength, and the fracture ductility, expressions that describe the cyclic strength coefficient and the cyclic strain-hardening exponent are established. By means of cyclic stress-strain curve, the limitations of the traditional methods of estimating the cyclic strength coefficient and the cyclic strain-hardening exponent are pointed out, and the ability of the new equations at describing the process are established. From the equations the cyclic strength coefficient and the cyclic strain-hardening exponent can be theoretically estimated using the monotonic ones. Furthermore, in the absence of the strength coefficient and the strain-hardening exponent, the cyclic ones can still be obtained from the expressions using the four conventional tensile performance parameters.

Keywords cyclic strain-hardening exponent, cyclic strength coefficient, strain-hardening exponent, strength coefficient, theoretical estimation

1. Introduction

When the fatigue behavior of metallic materials is being studied, especially the fatigue crack initiation life of metallic materials, it is necessary to know two material constants—the cyclic strength coefficient and the cyclic strain-hardening exponent (Ref 1-3). Without doubt, the best way to determine these two material constants is through comprehensive testing. However, fatigue tests are not only expensive but also time-consuming. Therefore, theoretically estimating the cyclic strength coefficient and the cyclic strain-hardening exponent is very useful.

Traditionally, there are three methods to theoretically estimate the cyclic strength coefficient and the cyclic strain-hardening exponent (Ref 4-8). As will be described in the following narrative, the first two methods use fatigue performance parameters, i.e., the fatigue strength coefficient, the fatigue strength exponent, the fatigue ductility coefficient, and the fatigue ductility exponent, while the third method makes use of the tensile performance parameters, i.e., the Young's modulus, the ultimate tensile strength, the fracture strength, and the fracture ductility. The estimated results from the first two methods have been discussed in Ref 4 and the conclusion is that the results are not satisfactory. Similarly, it will be seen that the estimated results from the third method are also not satisfactory.

Therefore, in the present article, equations that relate the cyclic strength coefficient and the cyclic strain-hardening exponent to the monotonic tensile ones are established using performance parameters from 22 alloys. Then, the formulas that express the strength coefficient and the strain-hardening exponent through these four conventional tensile performance parameters (the yield strength, the ultimate tensile strength, the fracture strength, and the fracture ductility) are used to develop

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Nomenclature	
K'	cyclic strength coefficient
K	strength coefficient
E	Young's modulus
σ_b	ultimate tensile strength
ε_f	fracture ductility
ψ	reduction of area
σ'_f	fatigue strength coefficient
b	fatigue strength exponent
$K'_{t1}, K'_{t2}, K'_{t3}$	theoretical cyclic strength coefficient
$n'_{t1}, n'_{t2}, n'_{t3}$	theoretical cyclic strain-hardening exponent
n'	cyclic strain-hardening exponent
n	strain-hardening exponent
$\sigma_{0.2}$	yield strength
σ_f	fracture strength
α	fracture ductility
N	crack initiation life
ε'_f	fatigue ductility coefficient
c	fatigue ductility exponent

expressions that describe the cyclic strength coefficient and the cyclic strain-hardening exponent. By means of the cyclic stress-strain curve, limitations of the third method are pointed out.

2. Traditional Methods Estimating the Cyclic Strength Coefficient and the Cyclic Strain-Hardening Exponent

Traditionally, there are three methods to theoretically estimate the cyclic strength coefficient and the cyclic strain-hardening exponent. The first method is (Ref 4, 5):

$$n' = 0.2 \quad K' = \frac{\sigma'_f}{\varepsilon_f'^{0.2}} \qquad \text{(Eq 1)}$$

The second method uses (Ref 4, 6, 7):

$$n' = \frac{b}{c} \quad K' = \frac{\sigma'_f}{\varepsilon_f'^{n'}} \qquad \text{(Eq 2)}$$

The third method is (Ref 4, 8):

$$n' = \frac{\lg\left(1.73\left(\frac{\sigma_f}{\sigma_b}\right)^{0.536}\right)}{\lg(36.2\varepsilon_f^{0.75}) - \lg\left(1 - 81.8\left(\frac{\sigma_b}{E}\right)\left(\frac{\sigma_f}{\sigma_b}\right)^{0.179}\right)} \qquad \text{(Eq 3)}$$

$$K' = \frac{0.933\sigma_b\left(\frac{\sigma_f}{\sigma_b}\right)^{0.74}}{0.125^{n'}\varepsilon_f^{0.75n'}} \qquad \text{(Eq 4)}$$

In Eq 1-4, K' refers to the cyclic strength coefficient, n' refers to the cyclic strain-hardening exponent, E is the Young's modulus, σ_b is the ultimate tensile strength, σ_f is the fracture strength and ε_f is the fracture ductility. In these equations, σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, ε'_f is the fatigue ductility coefficient and c is the fatigue ductility exponent.

In the first method described by Eq 1, expressions for σ'_f and ε'_f are (Ref 4):

$$\sigma'_f = 1.90\sigma_b \quad \varepsilon'_f = 0.758\varepsilon_f^{0.6} \qquad \text{(Eq 5)}$$

In the second method, Eq 2, the relationship among fatigue performance parameters and conventional tensile properties is (Ref 1, 4, 6, 9):

$$b = -0.0792 - 0.179\log\left(\frac{\sigma_f}{\sigma_b}\right) \qquad \text{(Eq 6)}$$

$$\sigma'_f = 1.18\sigma_b\left(\frac{\sigma_f}{\sigma_b}\right)^{0.946} \qquad \text{(Eq 7)}$$

$$c = -0.52 - \frac{1}{4}\lg\varepsilon_f + \frac{1}{3}\lg\left[1 - 82\left(\frac{\sigma_b}{E}\right)\left(\frac{\sigma_f}{\sigma_b}\right)^{0.179}\right] \qquad \text{(Eq 8)}$$

$$\varepsilon'_f = 0.538\varepsilon_f\left[1 - 82\left(\frac{\sigma_b}{E}\right)\left(\frac{\sigma_f}{\sigma_b}\right)^{0.179}\right]^{-1/3} \qquad \text{(Eq 9)}$$

Obviously, if conventional tensile performance parameters are known, the fatigue performance parameters can be estimated, and therefore the cyclic strength coefficient and the cyclic strain-hardening exponent can also be estimated.

In Ref 4, the estimated results from the first two methods have been discussed. It was pointed out that, on an average, the cyclic strength coefficient and the cyclic strain-hardening exponent estimated by the first method deviate from the experimentally determined ones by up to 41 and 86%, respectively (Ref 4). Those estimated using the second method deviate from the tested ones by 26 and 63%, respectively (Ref 4).

The first method (Eq 1) comes from following equations (Ref 4, 5, 7):

$$n' = 0.2 \qquad \text{(Eq 10)}$$

$$\sigma'_f = K'\varepsilon_f'^{n'} \qquad \text{(Eq 11)}$$

Equation 10 roughly expresses the cyclic strain hardening exponent. Equation 11 originates from the Ramberg–Osgood equation (Ref 10):

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \qquad \text{(Eq 12)}$$

where $\frac{\Delta\sigma}{2}$ and $\frac{\Delta\varepsilon}{2}$ are the cyclic stress amplitude and cyclic strain amplitude, respectively. The Ramberg–Osgood equation is a fitted equation using fatigue test data $\left(\frac{\Delta\sigma}{2}, \frac{\Delta\varepsilon}{2}\right)$, and deviation problems are inevitable when it is used at specific point (e.g., $\sigma'_f, \varepsilon'_f$). Therefore, Eq 11 may not precisely reflect the relationship between the fatigue strength coefficient σ'_f and the fatigue ductility coefficient ε'_f.

In addition, Eq 5, which is used in Eq 1, is deduced from the Manson–Coffin's equation (Ref 6, 11):

$$\Delta\varepsilon = \frac{3.5\sigma_b}{E}(N)^{-0.12} + \varepsilon_f^{0.6}(N)^{-0.6} \qquad \text{(Eq 13)}$$

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}(2N)^b + \varepsilon'_f(2N)^c \qquad \text{(Eq 14)}$$

Here N represents the fatigue crack initiation life. While Eq 13 roughly describes the fatigue behavior of metallic materials, Eq 5 expresses the relationship among the ultimate tensile strength, the fracture ductility, the fatigue strength coefficient, and the fatigue ductility coefficient. Therefore, it is not unusual that the cyclic strength coefficient and the cyclic strain-hardening exponent, estimated by the first method, deviate, on an average, from the experimentally determined ones by 41 and 86%, respectively.

In regard to the second method: on the one hand, Eq 2 only approximately represents the relationship among the cyclic strain-hardening exponent, the fatigue strength exponent, and the fatigue ductility exponent, while on the other hand, Eq 11 may not precisely reflect the correlation between the fatigue strength coefficient and the fatigue ductility coefficient. Thus, it is easy to understand that, on average, the cyclic strength coefficient and the cyclic strain-hardening exponent, estimated using the second approach, deviate from the experimental ones by 26 and 63%, respectively (Ref 4).

Now, in order to examine the precisions of the third method, Table 1-3 give the experimentally determined performance parameters of 22 alloys (Ref 1, 12-15). In these tables, K' and n' are established from test results, while K'_{t3} and n'_{t3} are derived from Eq 3-4, and are, hereafter, referred to as the theoretical cyclic strength coefficient and theoretical cyclic strain-hardening exponent, respectively. Therefore, $\delta K'_{t3} = \left(K'_{t3} - K'\right)/K'$ and $\delta n'_{t3} = \left(n'_{t3} - n'\right)/n'$.

Table 1 Performance parameters and the theoretical results for aluminum alloys (Ref 1, 12-15)

Material	LY12CZ (rod)	LY12CZ (plate)	LC4CS	LC9CGS3	2024-T4	7075-T6	2014-T6
E	73160	71022	72572	72180	70329	71018	68950
ε_f, %	18	30.2	18	28.3	43	41	29
α, %	3.0	8.0	3.0	6.0	15.1	13.5	7.25
σ_f	643.4	618.0	710.6	748.5	634	745	627
σ_b	545.1	475.6	613.9	560.2	476	579	510
$\sigma_{0.2}$	399.5	331.5	570.8	518.2	303	469	462
$\sigma_b/\sigma_{0.2}$	1.36	1.43	1.08	1.08	1.57	1.23	1.10
$\sigma_f/\sigma_{0.2}$	1.61	1.86	1.24	1.44	2.09	1.59	1.36
σ_f/σ_b	1.18	1.30	1.16	1.34	1.33	1.29	1.23
K	849.8	545.2	775.1	724.6	807	827	713.8
n	0.158	0.0889	0.063	0.071	0.2	0.113	0.091
K'	870.5	645.8	949.6	905.9	764	1090	682.1
n'	0.097	0.0669	0.08	0.101	0.08	0.130	0.072
K'/K	1.02	1.18	1.23	1.25	0.95	1.32	0.96
n'/n	0.61	0.75	1.27	1.42	0.4	1.15	0.79
K'_{t1}	1030.9	547.2	920.8	843.6	968.4	997.8	826.8
$\delta K'_{t1}$, %	18.4	−15.3	−3.0	−6.9	26.7	−8.5	21.2
K'_{t2}	1009.3	426.3	938.5	887.2	877.4	993.7	650.9
$\delta K'_{t2}$, %	15.9	−34.0	−1.8	−2.1	14.8	−8.8	−4.6
K'_{t3}	783.1	741.3	858.6	893.6	734.0	847.4	955.6
$\delta K'_{t3}$, %	−10.3	14.8	−9.6	−1.4	−3.9	−22.3	40.1
n'_{t1}	0.106	0.0660	0.07	0.101	0.09	0.148	0.119
$\delta n'_{t1}$, %	9.3	−1.3	−12.5	0.0	12.5	13.8	65.3
n'_{t2}	0.103	0.0655	0.07	0.093	0.08	0.148	0.067
$\delta n'_{t2}$, %	6.2	−2.1	−12.5	−7.9	0.0	13.8	−6.9
n'_{t3}	0.0832	0.0844	0.0768	0.0815	0.0796	0.0721	0.181
$\delta n'_{t3}$, %	−14.2	26.2	−4	−19.3	−0.5	−44.5	151.4

Table 2 Performance parameters and the theoretical results for alloy steels (Ref 1, 12-15)

Material	20	GH4133	GH36	30CrMnSiNi2A	GC-4	30CrMnSiA	AISI4340	40_1 (a)	40_2 (a)	42CrMo
E	213000	199200	203000	200063	200455	203005	193060	209000	209000	212000
ε_f, %	96	34.25	38.57	74	63.32	77.27	84	93	102	66
α, %	59	10	12.3	39	28	41	47.9	56	65	32
σ_f	710	1550.6	1016.0	2600.5	3511.6	1795.1	1655	1050	1330	1825
σ_b	441	1202	952	1655.3	1875.3	1177.0	1241	621	931	1413
$\sigma_{0.2}$	262	791	628	1308.3	1513.2	1104.5	1179	345	883	1379
$\sigma_f/\sigma_{0.2}$	2.71	1.96	1.62	1.99	2.32	1.63	1.40	3.04	1.51	1.32
$\sigma_b/\sigma_{0.2}$	1.68	1.52	1.52	1.27	1.24	1.07	1.05	1.8	1.05	1.02
K	738	…	…	2355.4	3150.2	1475.8	1579	…	1172	…
n	0.19	…	…	0.091	0.1468	0.063	0.066	0.22	0.06	0.05
K'	772	2024.3	1394.6	2647.7	3411.4	1771.9	1889	…	1434	…
n'	0.18	0.1098	0.0734	0.13	0.14	0.13	0.14	0.18	0.14	0.17
K'/K	1.05	…	…	1.12	1.08	1.20	1.20	…	1.22	…
n/n'	1.06	…	…	0.7	1.05	0.48	0.47	1.22	0.43	0.3
K'_{t1}	864.3	…	…	2703.3	3376.9	1820.8	1935.0	…	1461.9	…
$\delta K'_{t1}$, %	12.0	…	…	2.1	−1.0	2.8	2.4	…	1.9	…
K'_{t2}	320.7	2260.5	1453.8	2480.5	3259.1	2083.4	1943.2	623.3	1569.7	2184.4
$\delta K'_{t2}$, %	−58.5	11.7	4.2	−6.3	−4.4	17.6	2.9	…	9.5	…
K'_{t3}	912	1846.2	1936.9	2872.3	3920	2016.9	1808.6	1367	1640	1638
$\delta K'_{t3}$, %	18.1	−8.8	38.9	8.5	14.9	13.8	−4.3	…	14.6	…
n'_{t1}	0.19	…	…	0.13	0.14	0.11	0.188	0.18	0.13	0.17
$\delta n'_{t1}$, %	5.6	…	…	0.0	0.0	−15.4	34.3	0.0	−7.1	0.0
n'_{t2}	0.12	0.0943	0.0733	0.13	0.12	0.14	0.1478	0.11	0.14	0.15
$\delta n'_{t2}$, %	−33.3	−14.1	−0.1	0.0	−14.3	7.7	5.6	−38.9	0.0	−11.8
n'_{t3}	0.21	0.0847	0.2055	0.0732	0.0741	0.0818	0.0715	0.22	0.18	0.17
$\delta n'_{t3}$, %	16.7	−22.9	180	−43.7	−47.1	−35.6	−48.9	22.2	28.9	0.0

(a) 1 and 2 represent the different heat-treated conditions

Table 3 Performance parameters and the theoretical results for titanium alloys (Ref 1, 12-15)

Material	$BT20_{L1}$ (a)	$BT20_{T1}$ (a)	$BT20_{T2}$ (a)	Ti-6Al-4V	Ti-8Mo-1Mo-1V
E	115700	121000	118300	117215	117215
ε_f, %	51.6	51.6	42.77	53	66
α, %	20.8	20.8	14.9	21.7	31.7
σ_f	1309	1349.7	1261.2	1717	1565
σ_b	933	962	935.6	1234	1020
$\sigma_{0.2}$	867	902	886.3	1186	1007
$\sigma_b/\sigma_{0.2}$	1.08	1.07	1.06	1.04	1.01
$\sigma_f/\sigma_{0.2}$	1.51	1.50	1.42	1.45	1.55
K	974.8	1017	…	1545	1575
n	0.021	0.021	…	0.053	0.078
K'	1340.5	1541.3	1687.6	2106	1764
n'	0.105	0.114	0.107	0.162	0.14
K'/K	1.38	1.52	…	1.36	1.12
n/n'	0.20	0.18	…	0.33	0.56
K'_{t1}	1205.7	1262.4	…	1897.7	1930.6
$\delta K'_{t1}$, %	-10.0	-18.1	…	−9.9	9.4
K'_{t2}	1584.3	1644.6	1666.5	2082.7	1951.2
$\delta K'_{t2}$, %	18.2	6.7	−1.3	−1.1	10.7
K'_{t3}	1465.7	1546.7	1582.0	1851.4	1731.3
$\delta K'_{t3}$, %	9.3	0.4	−6.3	−12.1	1.8
n'_{t1}	0.0484	0.0489	…	0.130	0.144
$\delta n'_{t1}$, %	−53.9	−57.1	…	−19.7	2.9
n'_{t2}	0.1559	0.1503	0.081	0.154	0.138
$\delta n'_{t2}$, %	48.5	31.8	−23.9	4.9	−1.1
n'_{t3}	0.1697	0.1710	0.1728	0.0566	0.0712
$\delta n'_{t3}$, %	61.6	50.0	61.5	−65.1	−49.1

(a) 1 and 2 represent the different heat-treated conditions, L and T represent that the specimens are taken along and perpendicular the rolling directions, respectively

Results listed in the tables show $-22 < \delta K'_{t3} < 40\%$ and $-65 < \delta n'_{t3} < 180\%$. The absolute mean values $\overline{|\delta K'_{t3}|}$ and $\overline{|\delta n'_{t3}|}$ are 13 and 45%, respectively. The absolute mean values $\overline{|\delta K'_{t3}|}$ and $\overline{|\delta n'_{t3}|}$ show that, compared to the first two methods, the third method to theoretically estimate the cyclic strength coefficient and the cyclic strain-hardening exponent is the best. However, the cyclic strain amplitude $\Delta\varepsilon/2$ is determined by the cyclic strength coefficient and by the cyclic strain-hardening exponent, and the fatigue crack initiation life N is determined by the cyclic strain amplitude $\Delta\varepsilon/2$. Therefore, the third method described by Eq 3-4 should further improve upon the theoretical estimate of the cyclic strength coefficient and the cyclic strain-hardening exponent.

3. Equations Relating the Cyclic Strength Coefficient and the Cyclic Strain-Hardening Exponent to the Monotonic Tensile Ones

3.1 Equations Relating the Cyclic Strength Coefficient to the Monotonic Tensile One

Figure 1 shows the relationship between the cyclic strength coefficient and the monotonic tensile one. In this figure, one point (K, K') corresponds to one material, and K, K' are from Table 1-3 (Ref 1, 12-15). Fitting the points in Fig. 1 using the least-square method, the desired relationship can be obtained, i.e.,

$$K' = 57K^{0.545} - 1220 \quad \text{(Eq 15)}$$

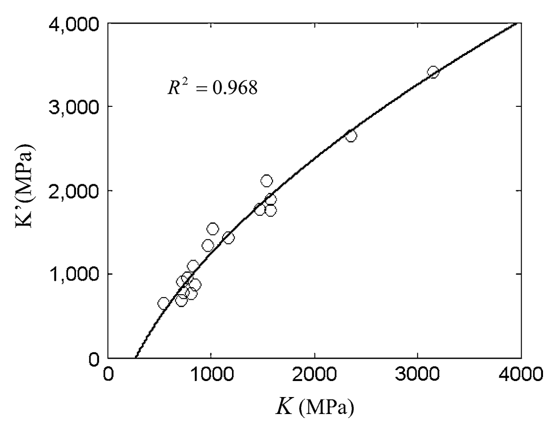

Fig. 1 Relationship between the cyclic strength coefficient and the monotonic tensile one

If the strength coefficient is known, the cyclic strength coefficient can be estimated by this relationship. As a display of "correctness" of Eq 15, the cyclic strength coefficients estimated from its use are denoted as K'_{t1}, and these values are listed in the tables. The relative deviation is defined as $\delta K'_{t1} = (K'_{t1} - K')/K'$.

Results listed in the tables show $-18\% \le \delta K'_{t1} \le 27\%$ and the absolute mean value $\overline{|\delta K'_{t1}|}$ equal to 10%. The reasons why the theoretical cyclic strength coefficient deviates from the experimental one may be that the strength coefficient itself is an experimental result, and there exists a difference between the test result and the true value. What is more, Eq 15 is fit to the points (K, K'), and deviation problem arises when it is used at specific point.

3.2 Equations Relating the Cyclic Strain-Hardening Exponent to the Monotonic Tensile One

To search for a relationship between the cyclic strain-hardening exponent and the monotonic tensile one, 22 alloys performance parameters listed in Table 1-3 were studied carefully. The performance parameters show the following three characteristics:

(1) $n' > n$ for $\alpha < 20\%$ and for $\sigma_f/\sigma_{0.2} < 1.6$.
(2) $n' < n$ for $\alpha < 20\%$ and for $\sigma_f/\sigma_{0.2} > 1.6$.
(3) $\frac{\sigma_f}{\sigma_{0.2}} - \frac{\sigma_b}{\sigma_{0.2}} \approx \frac{n}{n'}$ for $\alpha > 20\%$.

Considering these three characteristics the following relationships can be worked out:

$$n' = 1.06n\left(1 + \beta\left|1 - \frac{\sigma_b}{\sigma_{0.2}}\right|\right) \quad \text{(Eq 16)}$$

for $\alpha < 5\%$ or for $10\% \le \alpha < 20\%$;

$$n' = 1.06n\left(1 + \beta\left|1 - \frac{\sigma_f}{\sigma_b}\right|\right) \quad \text{(Eq 17)}$$

for $5\% < \alpha < 10\%$ and

$$n' = \frac{\sigma_{0.2}}{\sigma_f - \sigma_b} n \quad \text{(Eq 18)}$$

for $\alpha > 20\%$. In Eq 16-17, $\beta = 1$ for $\sigma_f/\sigma_{0.2} < 1.6$, but $\beta = -1$ for $\sigma_f/\sigma_{0.2} > 1.6$.

Results from Eq 16-18 are, for the time being, referred to as theoretical cyclic strain-hardening exponents and denoted as n'_{t1}. The difference between the theoretical cyclic strain-hardening exponent and the experimental one is defined as $\delta n'_{t1} = \left(n'_{t1} - n'\right)/n'$. The data listed in the tables show $-15\% \le \delta n'_{t1} \le 34\%$ and the absolute mean value $\left|\delta n'_{t1}\right|$ equal to 7%. The reasons for this may be that, on the one hand, both the strain-hardening exponent and the cyclic one are the fitted results of test data, so deviations between the fitted results and the "true" values are inevitable; on the other hand, 1.06, $\sigma_b/\sigma_{0.2}$, σ_f/σ_b and $\sigma_{0.2}/(\sigma_f - \sigma_b)$ are empirical factors for several alloys, which cannot precisely fit every alloy.

It is well known that both the cyclic strength coefficient and the cyclic strain-hardening exponent are from cyclic stress-strain curve, while the strength coefficient and the strain-hardening exponent are from monotonic tensile stress-strain curve. One tensile test can provide a monotonic tensile stress-strain curve, but only when a set of comprehensive fatigue tests are completed can cyclic stress-strain curves be obtained. Therefore, compared with the strength coefficient and the strain-hardening exponent, it is more difficult to obtain cyclic ones. However, Eq 15-18 approximately express the equations which relate the cyclic strength coefficient and the cyclic strain-hardening exponent to the monotonic tensile ones, so the cyclic strength coefficient and the cyclic strain-hardening exponent can be theoretically estimated when the tensile values are known. Then, the question becomes how to estimate the cyclic strength coefficient and the cyclic strain-hardening exponent in the absence of the tensile values?

4. The Cyclic Strength Coefficient and the Cyclic Strain-Hardening Exponent from Conventional Tensile Performance Parameters

The authors have previously established formulas to relate the strength coefficient and the strain-hardening exponent to four conventional tensile performance parameters—the yield strength, the ultimate tensile strength, the fracture strength, and the fracture ductility, i.e. (Ref 16-18)

$$K = \sigma_f \varepsilon_f^{-n} \quad \text{(Eq 19)}$$

$$n = \frac{\lg\left(\frac{\sigma_f^3 \sigma_b^2}{\sigma_{0.2}^5}\right)}{3\lg(500\varepsilon_f)} \quad \text{(Eq 20)}$$

for $\alpha < 5\%$ or for $10\% \le \alpha < 20\%$ and

$$K = \frac{\sigma_f \sigma_{0.2}}{\sigma_b} \varepsilon_f^{-n} \quad \text{(Eq 21)}$$

$$n = \frac{\lg\left(\frac{\sigma_f^2}{\sigma_{0.2}\sigma_b}\right)}{2\lg(500\varepsilon_f)} \quad \text{(Eq 22)}$$

for $5\% < \alpha < 10\%$ or for $\alpha > 20\%$.

The value for α is defined as the fracture ductility parameter: (Ref 16-18):

$$\alpha = \varepsilon_f \psi \quad \text{(Eq 23)}$$

Now, if Eq19-22 are substituted into Eq 15-18, then:

$$K' = 57\left(\sigma_f \varepsilon_f^{-n}\right)^{0.545} - 1220 \quad \text{(Eq 24)}$$

$$n' = 1.06\left(1 + \beta\left|1 - \frac{\sigma_b}{\sigma_{0.2}}\right|\right) \frac{\lg\left(\frac{\sigma_b^2 \sigma_f^3}{\sigma_{0.2}^5}\right)}{3\lg(500\varepsilon_f)} \quad \text{(Eq 25)}$$

for $\alpha < 5\%$ or for $10\% \le \alpha < 20\%$;

$$K' = 57\left(\frac{\sigma_f \sigma_{0.2}}{\sigma_b} \varepsilon_f^{-n}\right)^{0.545} - 1220 \quad \text{(Eq 26)}$$

$$n' = 1.06\left(1 + \beta\left|1 - \frac{\sigma_f}{\sigma_b}\right|\right) \frac{\lg\left(\frac{\sigma_f^2}{\sigma_{0.2}\sigma_b}\right)}{2\lg(500\varepsilon_f)} \quad \text{(Eq 27)}$$

for $5\% < \alpha < 10\%$;

$$K' = 57\left(\frac{\sigma_f \sigma_{0.2}}{\sigma_b} \varepsilon_f^{-n}\right)^{0.545} - 1220 \quad \text{(Eq 28)}$$

$$n' = \frac{\sigma_{0.2}}{\sigma_f - \sigma_b} \frac{\lg\left(\frac{\sigma_f^2}{\sigma_{0.2}\sigma_b}\right)}{2\lg(500\varepsilon_f)} \quad \text{(Eq 29)}$$

for $\alpha > 20\%$.

Equation 24-29 shows that if the four conventional tensile performance parameters are known, the cyclic strength coefficient and the cyclic strain-hardening exponent can be estimated. For the sake of demonstrating the correctness of Eq 24-29, the performance parameters listed in Table 1-3 are used to estimate the cyclic strength coefficient and the cyclic strain-hardening exponent. The results are tentatively known as the theoretical ones and are denoted as K'_{t2} and n'_{t2}, respectively. The relative deviations of K'_{t2} and n'_{t2} from the

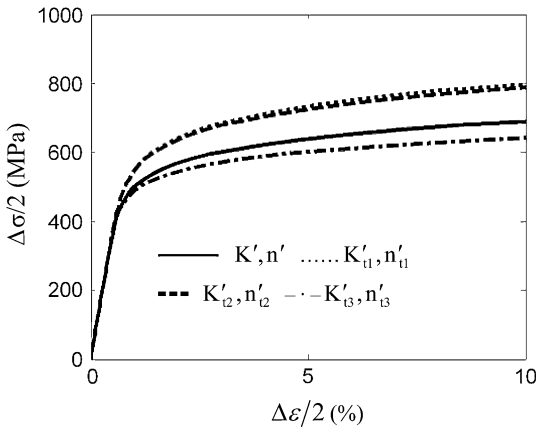

Fig. 2 Cyclic stress-strain curves for LY12CZ (rod) aluminum alloy

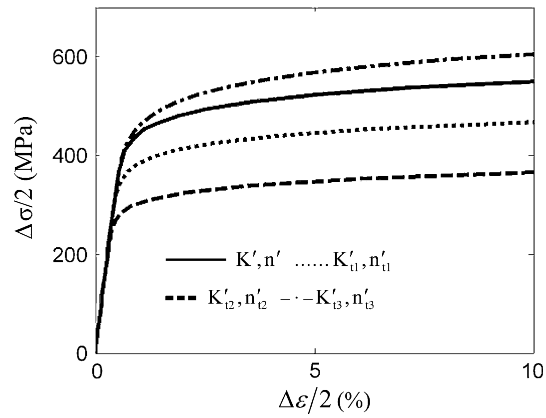

Fig. 3 Cyclic stress-strain curves for LY12CZ (plate) aluminum alloy

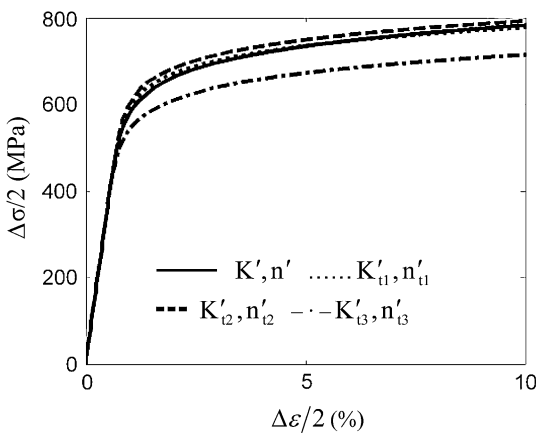

Fig. 4 Cyclic stress-strain curves for LC4CS aluminum alloy

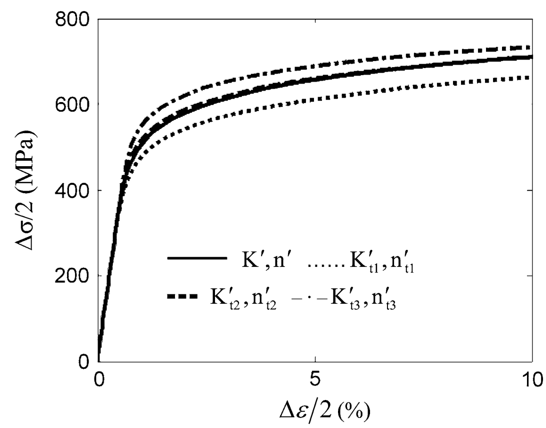

Fig. 5 Cyclic stress-strain curves for LC9CGS3 aluminum alloy

experimental ones are defined as $\delta K'_{t2} = \left(K'_{t2} - K'\right)/K'$ and $\delta n'_{t2} = \left(n'_{t2} - n'\right)/n'$. Results listed in the tables show $-59\% \leq \delta K'_{t2} \leq 18\%$, $-39\% \leq \delta n'_{t2} \leq 49\%$, and the absolute mean values $\overline{\left|\delta K'_{t2}\right|}$ and $\overline{\left|\delta n'_{t2}\right|}$ equal to 12 and 13%, respectively.

These results display that the theoretical cyclic strength coefficients and the theoretical cyclic strain-hardening exponents from Eq 24 to 29 are not as satisfactory as those from Eq 15 to 18. As was pointed out in Ref 16-18, Eq 19-22 express approximately the relationship among the tensile performance parameters of metallic materials, and originate from the test data, so deviations of both the strength coefficient and the strain-hardening exponent from Eq 19 to 22 are inevitable, which further affect the precision of the values of the cyclic strength coefficient and the cyclic strain-hardening exponent through use of Eq 24-29. Nevertheless, the results from Eq 24 to 29 are better than those from Eq 1 to 4. That is to say, in the absence of the strength coefficient and the strain-hardening exponent, Eq 24-29 provide another method to theoretically estimate the cyclic strength coefficient and the cyclic strain-hardening exponent using only tensile performance parameters.

It was found that to obtain the cyclic strength coefficient and the cyclic strain-hardening exponent, only four conventional tensile performance parameters are needed. It has been pointed out in Ref 16-18 that in order to get the strength coefficient and the strain-hardening exponent, a tensile stress-strain relation must be continuous, or the section of $\log \sigma - \log \varepsilon_p$ curve on the $\log \sigma$ axis must be determined. Compared with this, the determination of the four conventional tensile performance parameters is much easier, so the method to obtain the cyclic strength coefficient and the cyclic strain-hardening exponent through Eq 24-29 is simple and straightforward.

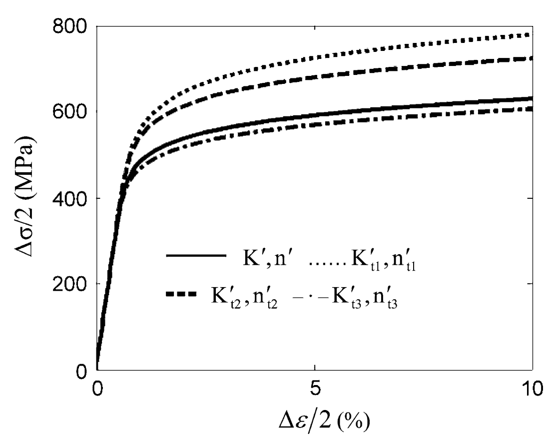

Fig. 6 Cyclic stress-strain curves for 2024-T4 aluminum alloy

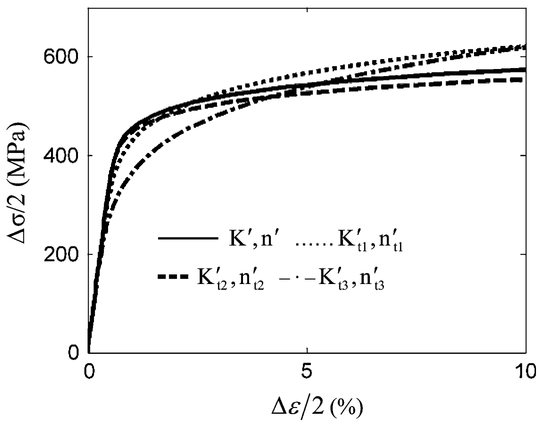

Fig. 7 Cyclic stress-strain curves for 2014-T6 aluminum alloy

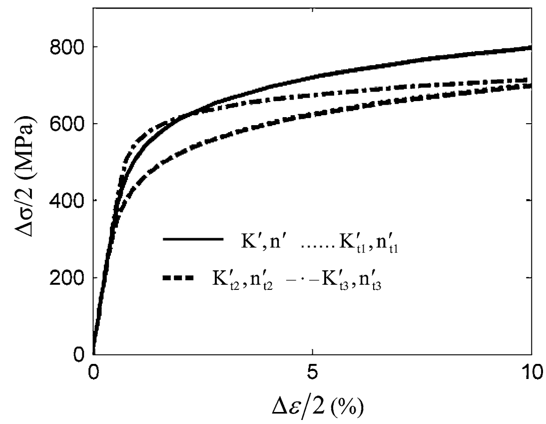

Fig. 8 Cyclic stress-strain curves for 7075-T6 aluminum alloy

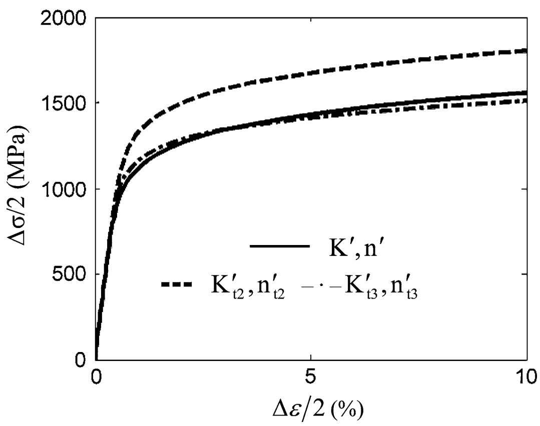

Fig. 9 Cyclic stress-strain curves for GH4133 alloy steel

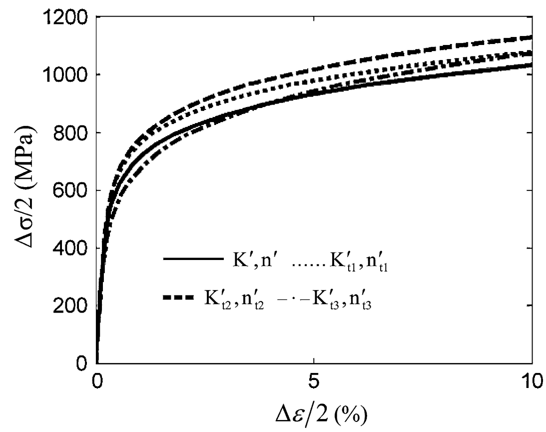

Fig. 10 Cyclic stress-strain curves for 40_2 alloy steel

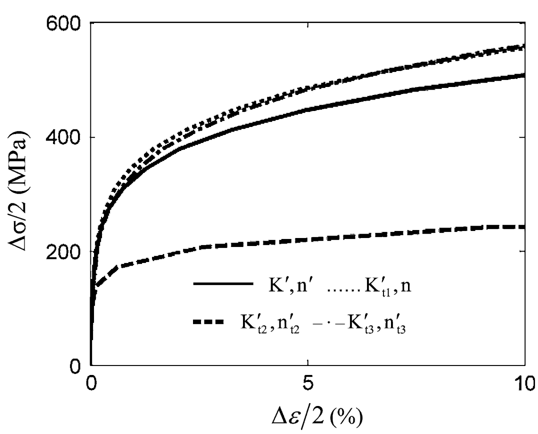

Fig. 11 Cyclic stress-strain curves for 20 alloy steel

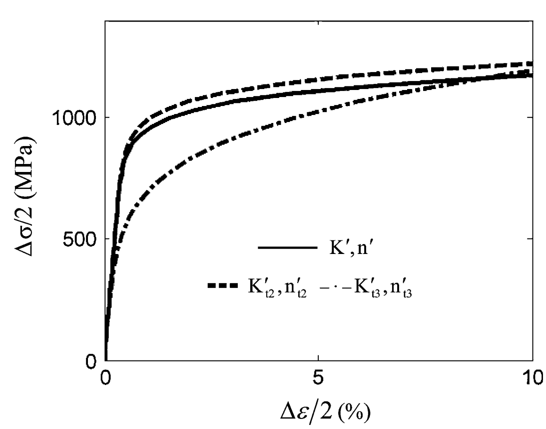

Fig. 12 Cyclic stress-strain curves for GH36 alloy steel

5. Theoretical Cyclic Stress-Strain Curves

The cyclic strength coefficient and the cyclic strain-hardening exponent listed in the tables are used in Eq 12, to develop the theoretical and the experimental cyclic stress-strain curves. These curves are shown in Fig. 2-21. In the figures, —, ····, — — — and –·– correspond to (K', n'), (K'_{t1}, n'_{t2}), (K'_{t2}, n'_{t2}) and (K'_{t3}, n'_{t3}), respectively. The figures show that:

(1) The methods described by Eq 15-18 and/or Eq 24-29 can be used to approximately estimate the cyclic strength coefficient and the cyclic strain-hardening exponent for aluminum alloys, alloy steels, and titanium alloys, while the method described by Eq 3-4 can be only used for aluminum alloys.

(2) The difference between the theoretical cyclic stress-strain curve and the experimental curve not only depends on the values of $\delta K'_{ti}$ and $\delta n'_{ti}$ (i = 1, 2, 3), but

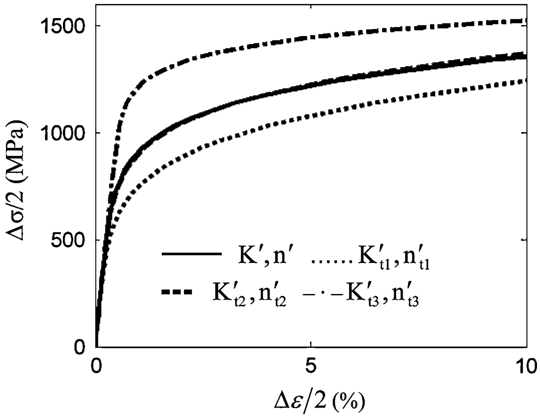

Fig. 13 Cyclic stress-strain curves for AISI4340 alloy steel

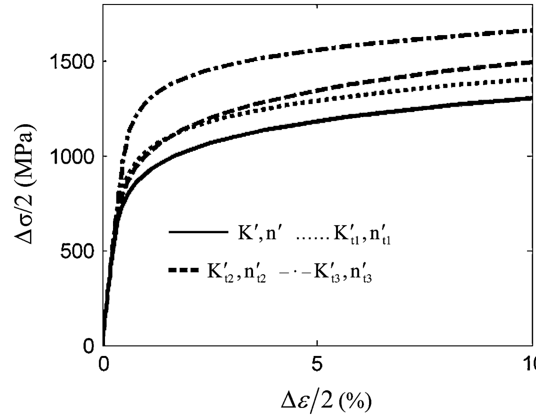

Fig. 14 Cyclic stress-strain curves for 30CrMnSiA alloy steel

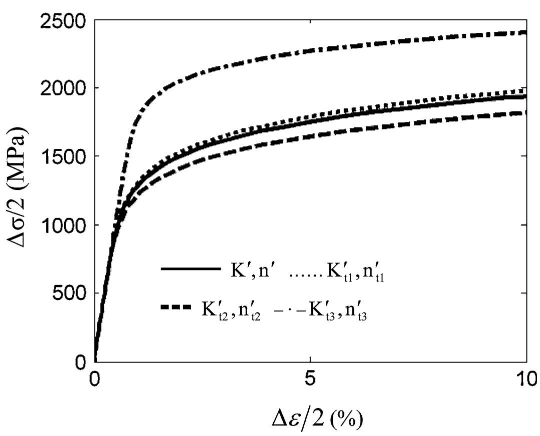

Fig. 15 Cyclic stress-strain curves for 30CrMnSiNi2A alloy steel

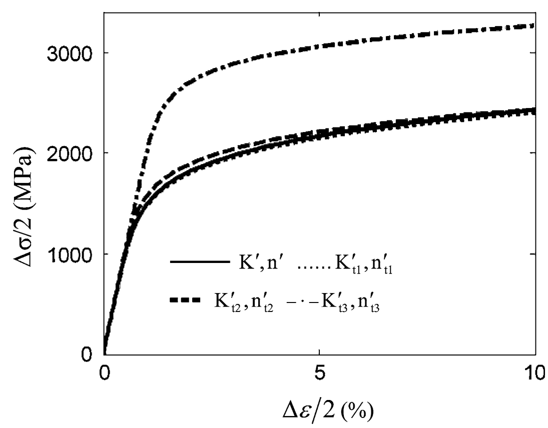

Fig. 16 Cyclic stress-strain curves for GC-4 alloy steel

also on the sign. When the signs are the same, the theoretical curves correlate well with the experimental ones. However, when the signs are not the same, the theoretical curves deviate greatly from the experimental ones. Take materials 7075-T6, GH4133 and 40_2 for examples. Although the $(\delta K'_{t3}, \delta n'_{t3})$ reach (−22.3, −44.5), (−8.8, −22.9), and (14.6, 28.9), respectively, the signs for them are the same, and the corresponding theoretical curves correlate well with the tested ones (Fig. 8-10). However, for materials 30CrMnSiA, 30CrMnSiNi2A, GC-4 and $BT20_{T2}$, the $(\delta K'_{t3}, \delta n'_{t3})$ are (13.8, −35.6), (8.5, −43.7), (14.9, −47.1), and (−6.3, 61.5), respectively, and for them the signs are not the same. As such, the corresponding theoretical curves deviate greatly from the experimental ones (Fig. 14-17).

(3) The effect of $\delta K'_{ti}$ (i = 1, 2, 3) to the theoretical cyclic stress-strain curve is not the same as that of $\delta n'_{ti}$ (i = 1, 2, 3). Take materials LY12CZ (plate) and 2014-T6, for examples, the $(\delta K'_{t1}, \delta n'_{t1})$ of them are (−15.3, −1.3) and (21.2, 65.3), respectively. It seems that the corresponding theoretical curve of LY12CZ (plate) should deviate less from the experimental one than that of 2014-T6, but they do not (Fig. 3 and 7). The reason for this phenomenon will be further studied.

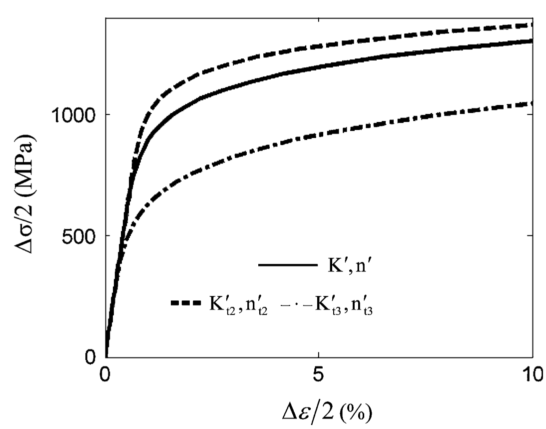

Fig. 17 Cyclic stress-strain curves for $BT20_{T2}$ titanium alloy

6. Conclusion

(1) For aluminum alloys, the expressions of the cyclic strength coefficient and the cyclic strain-hardening exponent are:

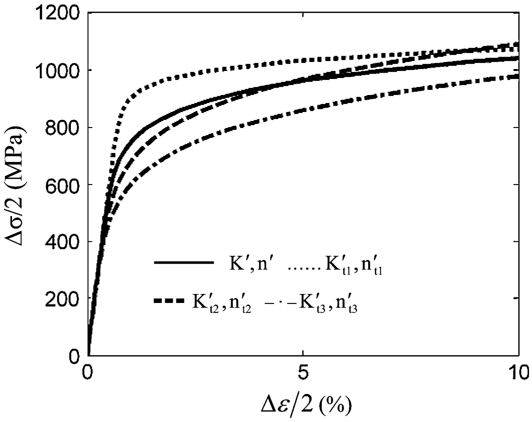

Fig. 18 Cyclic stress-strain curves for $BT20_{L1}$ titanium alloy

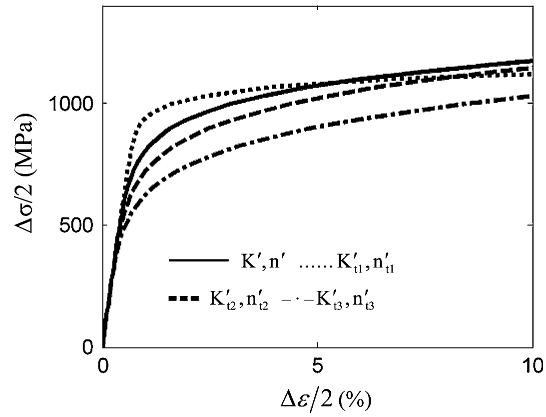

Fig. 19 Cyclic stress-strain curves for $BT20_{T1}$ titanium alloy

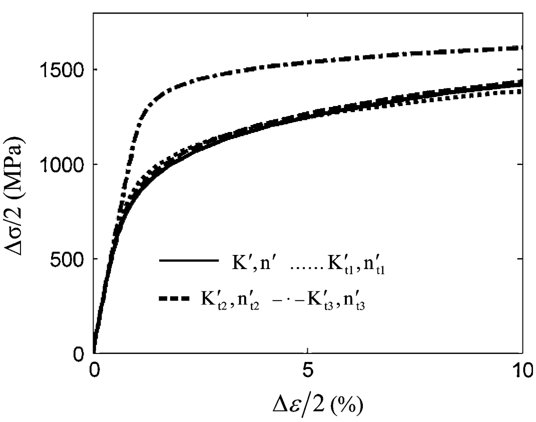

Fig. 20 Cyclic stress-strain curves for Ti-6Al-4V titanium alloy

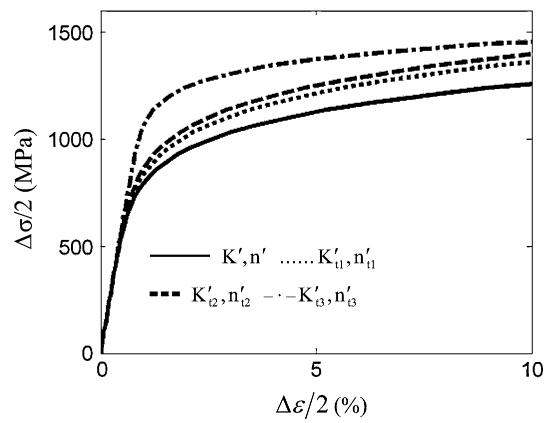

Fig. 21 Cyclic stress-strain curves for Ti-8Mo-1Mo-1V titanium alloy

$$n' = \frac{\lg\left(1.73\left(\frac{\sigma_f}{\sigma_b}\right)^{0.536}\right)}{\lg(36.2\varepsilon_f^{0.75}) - \lg\left(1 - 81.8\left(\frac{\sigma_b}{E}\right)\left(\frac{\sigma_f}{\sigma_b}\right)^{0.179}\right)}$$

$$K' = \frac{0.933\sigma_b\left(\frac{\sigma_f}{\sigma_b}\right)^{0.74}}{0.125^{n'}\varepsilon_f^{0.75n'}}$$

(2) For aluminum alloys, alloy steels, and titanium alloys, if the strength coefficient and the strain-hardening exponent are known, the expressions of the cyclic strength coefficient and the cyclic strain-hardening exponent are:

$$K' = 57K^{0.545} - 1220$$

$$n' = 1.06n\left(1 + \beta\left|1 - \frac{\sigma_b}{\sigma_{0.2}}\right|\right)$$

for $\alpha < 5\%$ or $10\% \le \alpha < 20\%$;

$$n' = 1.06n\left(1 + \beta\left|1 - \frac{\sigma_f}{\sigma_b}\right|\right)$$

for $5\% < \alpha < 10\%$;

$$n' = \frac{\sigma_{0.2}}{\sigma_f - \sigma_b}n$$

for $\alpha > 20\%$. Where $\beta = 1$ for $\sigma_f/\sigma_{0.2} < 1.6$ and $\beta = -1$ for $\sigma_f/\sigma_{0.2} > 1.6$.

(3) For aluminum alloys, alloy steels, and titanium alloys, in the absence of the strength coefficient and the strain-hardening exponent, the expressions of the cyclic strength coefficient and the cyclic strain-hardening exponent are:

$$K' = 57(\sigma_f\varepsilon_f^{-n})^{0.545} - 1220$$

$$n' = 1.06\left(1 + \beta\left|1 - \frac{\sigma_b}{\sigma_{0.2}}\right|\right)\frac{\lg\left(\frac{\sigma_b^2\sigma_f^3}{\sigma_{0.2}^5}\right)}{3\lg(500\varepsilon_f)}$$

for $\alpha < 5\%$ or for $10\% \le \alpha < 20\%$;

$$K' = 57\left(\frac{\sigma_f\sigma_{0.2}}{\sigma_b}\varepsilon_f^{-n}\right)^{0.545} - 1220$$

$$n' = 1.06\left(1 + \beta\left|1 - \frac{\sigma_f}{\sigma_b}\right|\right)\frac{\lg\left(\frac{\sigma_f^2}{\sigma_{0.2}\sigma_b}\right)}{2\lg(500\varepsilon_f)}$$

for $5\% < \alpha < 10\%$;

$$K' = 57\left(\frac{\sigma_f\sigma_{0.2}}{\sigma_b}\varepsilon_f^{-n}\right)^{0.545} - 1220$$

$$n' = \frac{\sigma_{0.2}}{\sigma_f - \sigma_b} \frac{\lg(\frac{\sigma_f^2}{\sigma_{0.2}\sigma_b})}{2\lg(500\varepsilon_f)}$$

for $\alpha > 20\%$.

The most important advantage of the above methods to obtain the cyclic strength coefficient and the cyclic strain-hardening exponent is that a comprehensive set of fatigue tests are not needed. If the strength coefficient and the strain-hardening exponent are known, the cyclic strength coefficient and the cyclic strain-hardening exponent can be estimated using the tensile analogs. However, in the absence of the strength coefficient and the strain-hardening exponent, the cyclic ones can be estimated using only the four conventional tensile performance parameters.

Acknowledgment

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